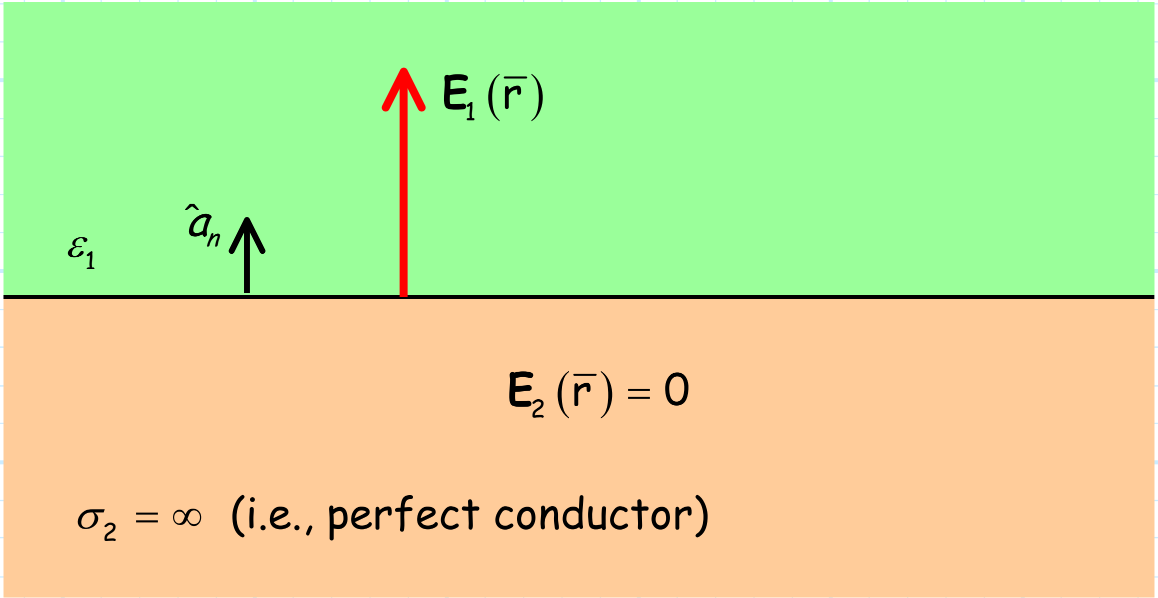
Boundary Conditions on Perfect Conductors

Consider the case where region 2 is a **perfect conductor**:



Recall $\mathbf{E}(\bar{r}) = 0$ in a perfect conductor. This of course means that **both** the tangential and normal component of $\mathbf{E}_2(\bar{r})$ are also equal to **zero**:

$$\mathbf{E}_{2t}(\bar{r}) = 0 = \mathbf{E}_{2n}(\bar{r})$$

And, since the **tangential** component of the electric field is **continuous** across the boundary, we find that **at the interface**:

$$\mathbf{E}_{1t}(\bar{r}_b) = \mathbf{E}_{2t}(\bar{r}_b) = 0$$

Think about what this means! The **tangential** vector component in the dielectric (at the dielectric/conductor boundary) is **zero**. Therefore, the electric field **at the boundary** only has a **normal** component:

$$\mathbf{E}_1(\bar{r}_b) = \mathbf{E}_{1n}(\bar{r}_b)$$

Therefore, we can say:

> The **electric field** on the **surface** of a **perfect conductor** is **orthogonal** (i.e., normal) to the conductor.

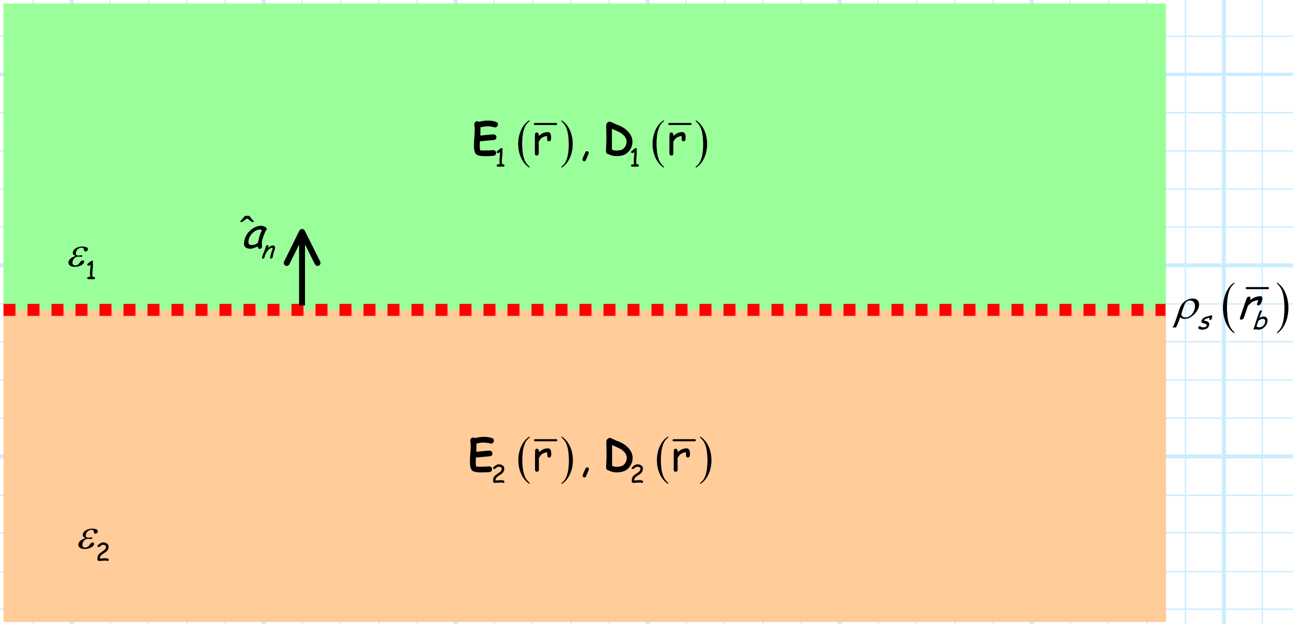
Q1: *What about the **electric flux density** $\mathbf{D}_1(\bar{r})$?*

A1: The relation $\mathbf{D}_1(\bar{r}) = \varepsilon_1 \mathbf{E}_1(\bar{r})$ is still of course valid, so that the **electric flux density** at the surface of the conductor must **also be orthogonal** to the conductor.

Q2: *But, we learned that the **normal** component of the **electric flux density** is **continuous** across an interface. If $\mathbf{D}_{2n}(\bar{r}) = 0$, why isn't $\mathbf{D}_{1n}(\bar{r}_b) = 0$?*

A2: Great question! The answer comes from a more **general** form of the **boundary condition**.

Consider again the interface of two **dissimilar dielectrics**. This time, however, there is some **surface charge distribution** $\rho_s(\bar{r}_b)$ (i.e., **free** charge!) **at the dielectric interface**:



The **boundary condition** for this situation turns out to be:

$$\hat{a}_n \cdot \left[\mathbf{D}_{1n}(\bar{r}_b) - \mathbf{D}_{2n}(\bar{r}_b)\right] = \rho_s(\bar{r}_b)$$
$$D_{1n}(\bar{r}_b) - D_{2n}(\bar{r}_b) = \rho_s(\bar{r}_b)$$

where $D_n(\bar{r}_b) = \hat{a}_n \cdot \mathbf{D}_n(\bar{r}_b)$ is the **scalar component** of $\mathbf{D}_n(\bar{r}_b)$ (note the **units** of each side are C/m^2!).

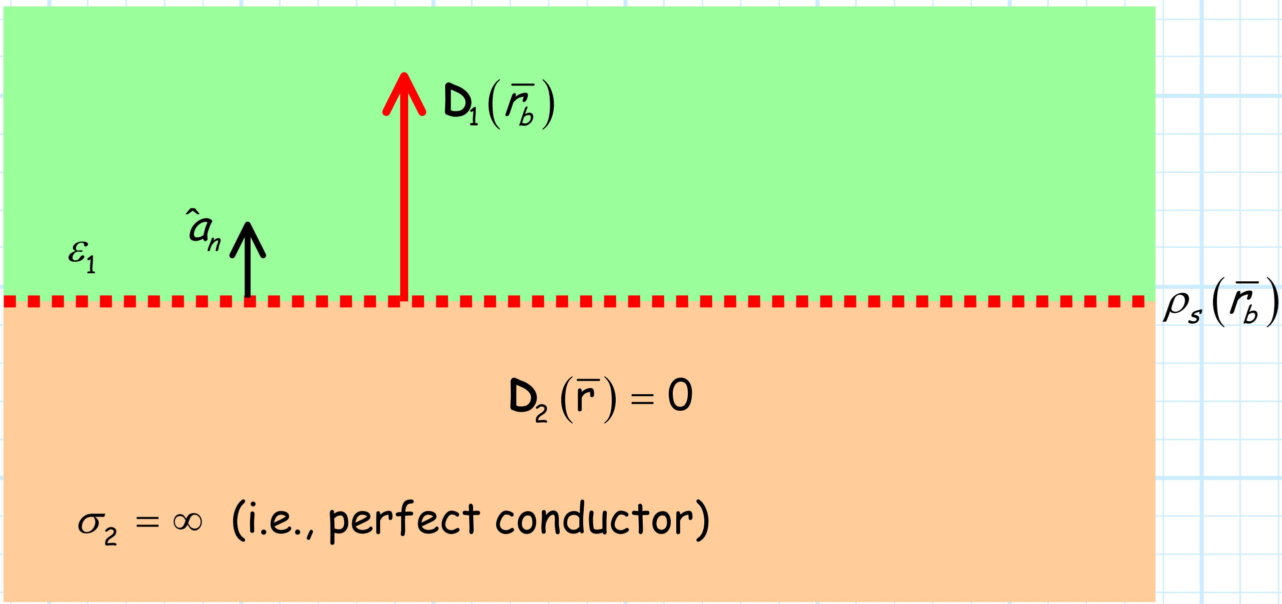
Note that **if** $\rho_s(\bar{r}_b) = 0$, this boundary condition returns (both physically and mathematically) to the case studied earlier—the **normal** component of the electric flux density is **continuous** across the interface.

This more **general** boundary condition is useful for the dielectric/**conductor** interface. Since $\mathbf{D}_2(\bar{\mathbf{r}}) = 0$ in the conductor, we find that:

$$\hat{a}_n \cdot \left[\mathbf{D}_{1n}(\bar{r}_b) - \mathbf{D}_{2n}(\bar{r}_b)\right] = \rho_s(\bar{r}_b)$$
$$\hat{a}_n \cdot \mathbf{D}_{1n}(\bar{r}_b) = \rho_s(\bar{r}_b)$$
$$D_{1n}(\bar{r}_b) = \rho_s(\bar{r}_b)$$

In other words, the **normal** component of the **electric flux density** at the **conductor surface** is equal to the **charge density** on the conductor surface.

Note in a perfect conductor, there is **plenty** of **free** charge available to form this charge density ! Therefore, we find in **general** that $D_{1n} \neq 0$ at the surface of a conductor.



Summarizing, the boundary conditions for the **tangential** components field components at a **dielectric/conductor** interface are:

$$\mathbf{E}_{1t}(\bar{r}_b) = 0$$

$$\mathbf{D}_{1t}(\bar{r}_b) = 0$$

but for the **normal** field components:

$$D_{1n}(\bar{r}_b) = \rho_s(\bar{r}_b)$$

$$E_{1n}(\bar{r}_b) = \frac{\rho_s(\bar{r}_b)}{\varepsilon_1}$$

Again, these boundary conditions describe the fields **at the conductor/dielectric interface**. They say **nothing** about the value of the fields at locations above this interface.